विद्या सवर्थि साधिका

ANANDALAYA
PRE- BOARD EXAMINATION
Class: XII

Subject: Mathematics — M.M : 80

Date : 06 -01-2023 — Time : 3 Hours

General Instructions:
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) - type questions of 2 mark each.
4. Section C has 6 Short Answer (SA) - type questions of 3 mark each.
5. Section D has 4 Long Answer (LA) - type questions of 5 mark each.
6. Section E has 3 sources based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions) Each question carries 1 mark

1. If A = $[a_{ij}]$ is a 2×3 matrix such that $a_{ij} = \begin{cases} i+j, & if\ i \geq j \\ i-j, & if\ i < j \end{cases}$ then $a_{12} =$ _______. (1)

A) 0 B) 3 C) 1 D) -1

2. If $|A| = 3\ and\ A^{-1} = \begin{bmatrix} 3 & -1 \\ -\frac{5}{3} & \frac{2}{3} \end{bmatrix}$, then $adj\ A =$ ______. (1)

A) $\begin{bmatrix} 9 & 3 \\ 5 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$ C) $\begin{bmatrix} \frac{1}{3} & -1 \\ -5 & 2 \end{bmatrix}$ D) $\begin{bmatrix} 2 & -3 \\ -5 & 9 \end{bmatrix}$

3. If $y = cos(sinx^2)$, find $\frac{dy}{dx}\ at\ x = \sqrt{\frac{\pi}{2}}$. (1)

A) 0 B) $\frac{\pi}{2}$ C) $\frac{1}{2}$ D) -1

4. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15\ then\ |\vec{x}| = ?$ (1)

A) 8 B) 4 C) 2 D) 1

5. If $\int e^x \left(\frac{x-1}{x^2}\right) dx = g(x) + C$, then $g(x) =$ _______. (1)

A) $\frac{-1}{x^2}$ B) $\frac{1}{x}$ C) $\frac{-1}{x}$ D) $\frac{1}{x}e^x$

6. Find the product of the order and degree of the following differential equation: (1)

$$x\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y^2 = 0$$

A) 4 B) 6 C) 2 D) 8

7. If $P(A) = 0.4, P(B) = p$ and $P(A \cup B) = 0.7$. Find the value of p, if A and B are independent events. (1)

A) 0.3 B) 0.2 C) 0.5 D) $\frac{1}{4}$

8. Evaluate: $\int_1^e \frac{1}{x\sqrt{1-(logx)^2}}\, dx$ (1)

A) 0 B) π C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$

9. Integrating factor of the differential equation $cos^2x\frac{dy}{dx} + y = tanx$ is ________. (1)

A) e^x B) e^{tanx} C) $tanx$ D) sec^2x

10. Evaluate : $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$ (1)

A) $\frac{\pi}{6}$ B) 0 C) 1 D) -1

11. For what value of *x*, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular? (1)

A) 3 B) 2 C) 6 D) $\frac{1}{3}$

12. A unit vector perpendicular to both of the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath}$. (1)

A) $\frac{\hat{\imath}}{\sqrt{2}} + \frac{\hat{\jmath}}{\sqrt{2}}$ B) $\frac{\hat{\imath}}{\sqrt{2}} - \frac{\hat{\jmath}}{\sqrt{2}}$ C) $\frac{-\hat{\imath}}{\sqrt{2}} + \frac{\hat{\jmath}}{\sqrt{2}}$ D) $\hat{\imath} + \hat{\jmath}$

13. Find $|\vec{a} \times \vec{b}|$ if $|\vec{a}| = 13$, $|\vec{b}| = 5$, $\vec{a}.\vec{b} = 60$. (1)

A) 5 B) 10 C) 25 D) 13

14. Find the value of k so that $f(x) = \begin{cases} \frac{x^2+3x-10}{x-2} & , \quad x \neq 2 \\ k & , \quad x = 2 \end{cases}$ is continuous at 2. (1)

A) 7 B) 10 C) 3 D) 14

15. Find the direction cosines of the line: $\frac{x-1}{2} = -y = \frac{z+1}{2}$ (1)

A) $\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$ B) $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$ C) $\frac{-2}{3}, \frac{1}{3}, \frac{-2}{3}$ D) $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$

16. If a matrix A = $\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A|^3 = 125$ then the value of α = ________ (1)

A) ± 3 B) ± 2 C) ± 5 D) 0

17. For maximum or minimum of objective function, the point lies _______ (1)

A) inside the feasible region B) at the boundary line of the feasible region

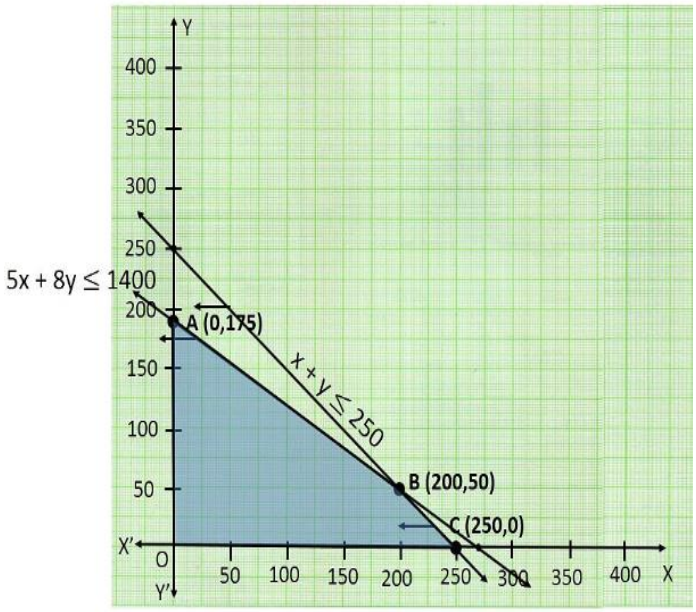
C) on the vertex point of the boundary of the feasible region D) none of these.

18. The corner points of the shaded bounded feasible region of an LPP are $A\,(0, 175)$, $B(200, 50)$, $C(250, 0)$, O(0,0) as shown in the figure. The maximum value of the objective function. (1)

Z = $4500\,x + 5000y$ occurs at_______.

A) (0,175) B) (250,0

C) (200,50) D) $(0, 0)$



ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

19. **Assertion (A):** If $A = [a_{ij}]$ is $m \times p$ matrix and $B = [b_{ij}]$ is $p \times n$ matrix then the product AB is a (1)

matrix of order $m \times n$.

Reason (R): $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $B = [2 \quad 3 \quad 4]$ .Then the product BA is of order 3×3.

20. **Assertion (A):** Angle between two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by $cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$. (1)

Reason (R): The angle between the lines $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$
$\vec{r} = 5\hat{\jmath} - 2\hat{k} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ is $\cos^{-1}\frac{19}{21}$.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Let $A = \{1, 2, 3\}$ and let $R_1 = \{(1,1), (1,3), (3,1), (2,2), (2,1), (3,3)\}$, check whether R_1 is equivalence relation or not? (2)

22. If $x = 10(t - sint)$, $y = 12(1 - cost), -\frac{\pi}{2} < t < \frac{\pi}{2}$. find $\frac{dy}{dx}$. (2)

23. An edge of a variable cube is increasing at the rate of 5cm per second. How fast is the volume increasing when the side is 15cm? (2)

24. If $\vec{p} = 5\hat{\imath} + \lambda\hat{\jmath} - 3\hat{k}$, $\vec{q} = \hat{\imath} + (\lambda - 3)\hat{\jmath} - 5\hat{k}$, then find the value of λ, so that $\vec{p} + \vec{q}$ *and* $\vec{p} - \vec{q}$ are perpendicular vectors. (2)

OR

If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, find $(\vec{r} \times \hat{\imath}).(\vec{r} \times \hat{\jmath}) + xy$.

25. Find the vector equation of the line passing through $A(1, 2, -1)$ *and* parallel to the line $5x - 25 = 14 - 7y = 35z$. (2)

SECTION C

This section comprises of short answer type questions (SA) of 3 marks each)

26. Evaluate: $\int \frac{1}{\sqrt{5-4x-2x^2}} dx$. (3)

OR

Using properties of integrals, evaluate : $\int_0^{\pi} \frac{x\, sinx}{1 + cos^2x} dx$

27. Solve the differential equation $2x^2\frac{dy}{dx} - 2xy + y^2 = 0$. (3)

28. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{3+sin2x} dx$ **OR** Evaluate: $\int_0^{\frac{\pi}{2}} \frac{cosx}{(1+sinx)(2+sinx)} dx$ (3)

29. Evaluate : $\int e^{3x} \, cosx\, dx$ (3)

OR

Using properties of integrals, evaluate : $\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1} + \sqrt[3]{11-x}} dx$

30. Solve the following problem graphically: Minimise and Maximise $Z = 3x + 9y$ subject to the constraints: $x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$, $x \geq 0, y \geq 0$. (3)

OR

A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹17.50 per package on nuts and ₹ 7 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day? Form the above as a linear programming problem and solve it graphically.

31. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die? (3)

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Using matrix method, solve the following system of equations: (5)

$$x - y + 2z = 7 \;;\quad 3x + 4y - 5z = -5; \quad 2x - y + 3z = 12$$

33. Let N be the set of natural numbers and let R be a relation on $N \times N$ defined by $(a, b)R(c, d) \Leftrightarrow ad = bc \; for \; all \; (a, b), (c, d) \in N \times N$. Show that R is an equivalence relation. (5)

OR

Check whether the relation R in R defined by $R = \{(a, b): a, b \in R, \quad a \leq b^3\}$ is reflexive, symmetric or transitive. Illustrate with examples.

34. Find the area bounded by the region : $\{(x, y) : \; x^2 + y^2 \leq 4, \; x + y \geq 2\}$ (5)

OR

Find the area of the region bounded by the parabola $y^2 = 4ax$ and its latusrectum.

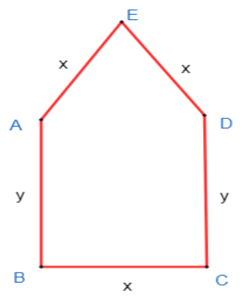
35. Find the shortest distance between the lines $\frac{x-5}{1} = \frac{y-4}{-2} = \frac{z-4}{1}$ and $\frac{x-1}{7} = \frac{y+2}{-6} = \frac{z+4}{1}$. (5)

SECTION E

This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts $(i), (ii), (iii)$ of marks 1, 1, , 2 respectively. The third case study question has two subparts of 2 marks each.

36. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, We need to find the dimensions of the rectangle that will produce the largest area of the window using second derivative test. (5)

Let x m be the side of the equilateral triangle, and y m. be the length of the rectangle and Area of the window is A sq. m.



i) Area of the window in terms of x, A = _________. (1)

ii) $\frac{dA}{dx}$ in terms of x = __________. (1)

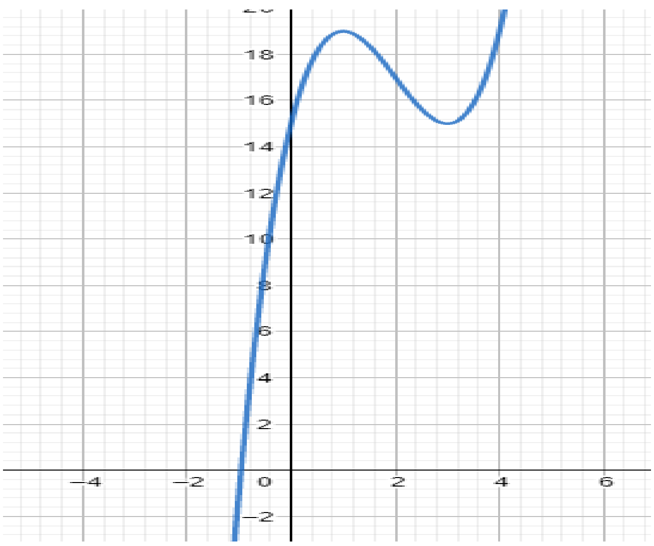
iii) Length of the side of the equilateral triangle (x) =_________ (2)

OR

Length of the rectangle (y) = _________

37 Given the graph of the function:

$$f(x) = x^3 - 6x^2 + 9x + 15.$$



i) Find the critical point of the function. (1)

ii) Find all the points of local maxima and local minima of the function. (1)

iii) Find local minimum or local maximum values. (2)

OR

Find the intervals in which the function $f(x) = x^3 - 6x^2 + 9x + 15$is strictly increasing/ strictly decreasing.

38 A problem in Mathematics is given to 3 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

Based on the above information and using the concept of independent events, answer the following. What is the probability that:

i) Problem is solved. (2)

ii) Exactly one of them will solve the problem. (2)